Research and Discussion Solution time comparisons between FFT and DFT

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Abstract:

Fast Fourier transform is advance technology for many engineering applicationThe aim of this paper is to study , analyize and enhance the solution time of DFT using FFT a logarithm and Excel software program . The parameter which were taken into consideration of the calculation are number of point , type of FFT after analysis and implementation of the logarithm ,the results were obtained in terms of chart for solution time versus number of point increased and average time for solving one point using DFT is 0.76050min while the average solution time using FFT is 0.38025 min the saved in time by using FFT is 0.38025 min. while the FFT solution time using MATLAB is real time solution in the instant of command.

Key words: DFT, FFT, a logarithm,MATLAB ,solution , time

المستخلص :

تحويل فوريه السريع هو تقنية متقدمة للعديد من التطبيقات الهندسية . الهدف من هذه الورقة هو دراسة وتحليل وتحسين وقت حل تحويل فورير المتقطع باستخدام خوارزميات تحويل فورير السريع وذلك باستخدام برنامج الاكسل والماتلاب . العوامل التي تم اخذها في الاعتبار عند الحساب هي عدد النقاط ونوع تحويل فورير السريع . بعد تحليل وتنفيذ اللوغريثم , تم الحصول علي النتائج في شكل رسم بياني لوقت الحل مقابل عدد النقاط . اظهرت النتائج انه كلما زاد عدد النقاط زاد وقت الحل , وان متوسط الوقت لحل نقطة واحدة باستخدام تحويل فوريه المتقطع هو 0.765050 دقيقةبينما متوسط وقت الحل باستخدام تحويل فورير السريع هو0.38025 دقيقة , عليه فان الوق التي تم توفيره باستخدام تحويل فورير السريع هو 0.38025 دقيقة. بينما وقت الحل باستخدام برنامج الماتلاب هو حل في الوقت الفعلي في لحظة الأمر .

الكلمات المفتاحية: تحويل فورير المتقطع، تحويل فورير السريع، الخوارزمية، الماتلاب، زمن الحل.

1- Literature review

Fourier Transform [1-6] has long been established as an instrumental tool applied in electrical signal spectrum and filter analysis, sampling and series, antenna, television image convolution as well as radio broadcasting (7). Being the limiting case of Fourier Series for non-periodic signals, FT is used to convert signal to frequency domain as the frequency domain has many superlative benefits especially for analytical purposes rather than in the classical time domain. In order to solve different various problems especially in digital image processing, the disceret version of FT, typically regarded as Discrete Fourier Transform (DFT) has been formulated.

Due to the large number of discrete samples required for DFT operation which requires complex multiplications, Fast Fourier Transform (FFT) has been introduced to significantly reduce the computational complexity by just requiring $N\log N$multiplications for N samples. This computational complexity issue becomes worse when higher dimensional signals such as image signals, which are two dimensional, are represented and processed using FFT. To further reduce the computational complexity and solution time .

FFT is a useful signal representation method that can be applied for fast processing especially when the signals are two dimensional like image signals. The image signals can be transformed using FFT in image processing since the signals are typically structured, rendering a sparse spectrum (8). Although FFT is known to has reduced computational complexity than DFT, and hence reducing time of solution .Various FFT algorithms have been proposed in literature to reduce the computational complexity, as presented in . One of the approaches which have been widely referred to in literature is the sparse FFT(sFFT) presented in [9]. However this method is formulated for one-dimensional case only. Therefore signals which are two dimensional such as image signals are unable to be represented using a one-dimensional sFFT model. The many image processing applications such as lithography, medical imaging, evolutionary arts and particle detection in wastewater treatment are infeasible to be applied with a one-dimensional sFTT.

A multi-dimensional sFFT has been proposed in (8) to cater the multi-dimensional cases. This multi-dimensional sFFT, including the 2D sFFT, has been widely studied and applied in a number of fields such as radar signal processing, lithography illumination and deep learning .

2- significance of the research :

The importance of this paper is more convenient to the application of digital signal processing specially for speech and image signal processing to reduce the number of addition and multiplication with low complexity .

3-problem statement :

The conventional discrete Fourier transform are used to analyize the digital image take long time to process the digital image processing for high order more than 8 .

4-objective of the paper :

. To study discrete Fourier transform

. To analyize the fast fourier transform a logarithm .

. To calculate the solution time for discrete and fast fourier transform .

. To compare the solution time between discrete and fast fourier transform .

5-methodology :

5.1 Description:

The solution time for different high order of number point N=2,4,8,16,32,64,128,256,512 and 1024using discrete fourier and fast fourier transform was measured and plotted versus number of point.

The conventional Discrete Fourier Transform (DFT) is given by :

$$X(n)=\sum_{k=0}^{N-1} x_0(k)e^{-j2\pi nk/N} \qquad n=0,1,\ldots,N-1$$

(1)

and the inverse Discrete Fourier Transform (IDFT) is given by:

$$x(k)=\sum_{n=0}^{N-1} X_0(n)e^{j2\pi nk/N} \qquad k=0,1,\ldots,k-1$$

$$X(k)=\sum_{n=0}^{N-1} x[n]W^{-kn}$$

(2) (3)

$$X(k)=\sum_{neven} x[n]W^{-kn}+\sum_{nodd} x[n]W^{-kn}$$

(4)

$$X(k)=\sum_{m=0}^{N/2-1} x[2m]W^{-2mk}+\sum_{m=0}^{N/2-1} x[2m+1]W^{-k(2m+1)}$$

(5)

$$X(k)=\sum_{m=0}^{N/2-1} x[2m]W_{N/2}^{mk}+W_N^k\left(\sum_{m=0}^{N/2-1} x[2m+1]W_{N/2}^{mk}\right)$$

(6)

$$W_N^{2mk}=W_{N/2}^{mk}$$

(7)

$$W_{N/2}^{m+N/2}=W_{N/2}^{m}W_{N/2}^{N/2}=W_{N/2}^{m}$$

(8)

$$W_N^N=e^{-2\pi}=\cos(-2\pi)-j\sin(-2\pi)=1$$

(9)

$$W_N^{N/2}=-1$$

(10)

And for FFT alogarithm is given by :

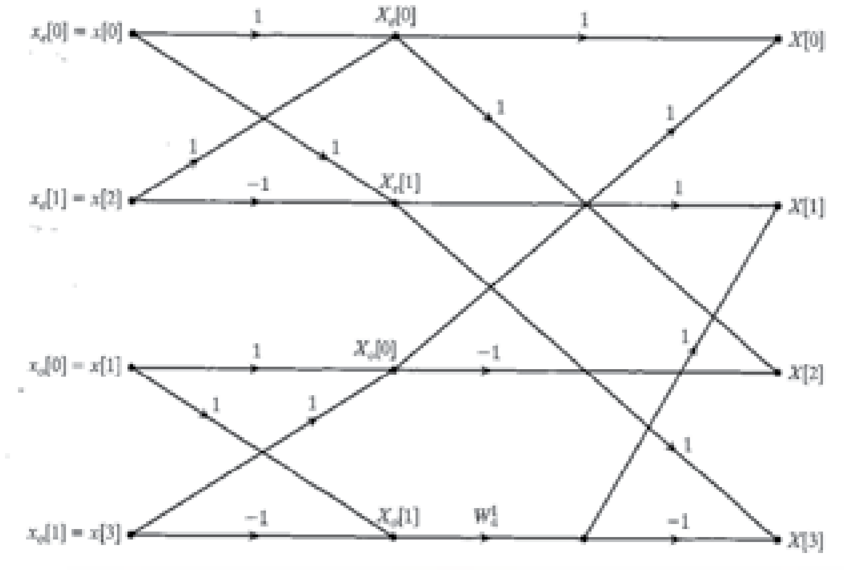


Figure 1:FFT block diagram for 4point

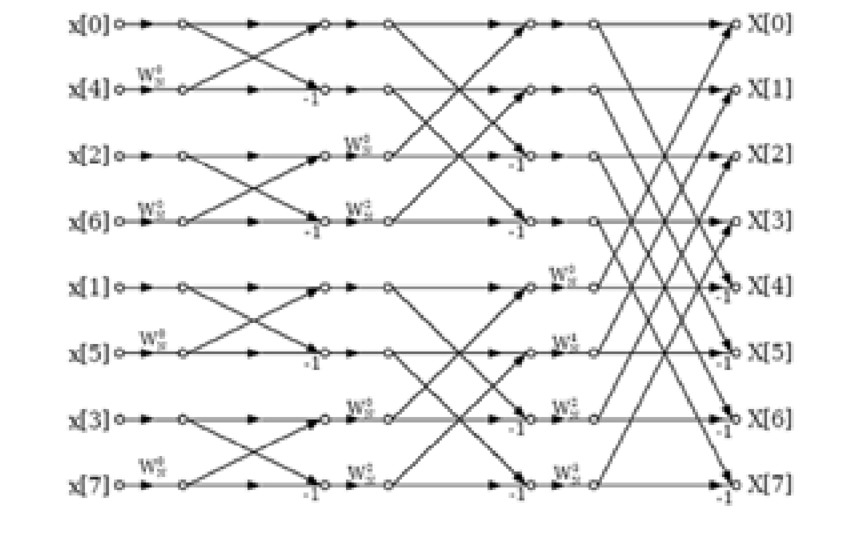


Figure 2:FFT block diagram for 8point

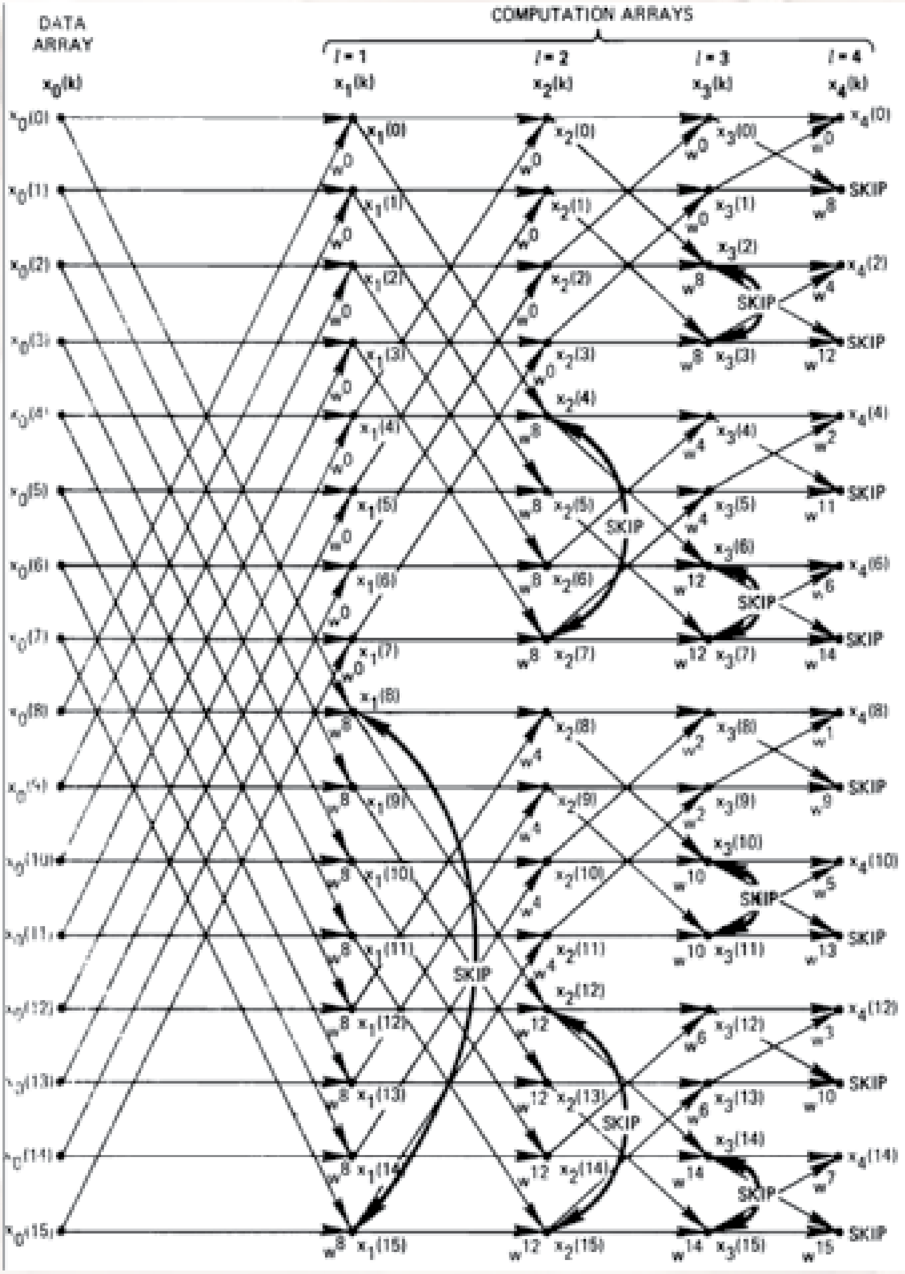


Figure 2:FFT block diagram for 16point

5.3- Environment parameter

Parameter	Value
Problem point (N)	2,4,8,16,….,1024
Type of FFT	Time decimation and frequency decimation

5.4 Implementation:

The environment parameter is implemented using excel software program to calculate DFT and FFT as show below :

$N = 2 \quad \Rightarrow X(n) = [1,0]$

$$X(n) = \sum_{k=0}^{1} x(k)e^{-j2\pi nkl2} \qquad (11)$$

$$X(0) = x(0)e^{0} + x(1)e^{-j\pi(0)} = x(0) + x(1) = 1 + 0 = 1$$

$$X(1) = x(0)e^{0} + x(1)e^{-j\pi} = 1 + 0(\cos\pi - j\sin\pi) = 1 - (-1) = 1$$

i.e Time domain $N = 2 \Rightarrow X(n) = [1,0]$ &frequency domain

i.e $K = 2 \Rightarrow X(K) = [1,1]$

the time soltion of 2 point usin DFT IS 3 min while for FFT is 6 min

$N = 4 \Rightarrow X(n) = [1,1,0,2]$

$$X(n) = \sum_{k=0}^{3} x(k)e^{-j\pi nkl2}$$

(12)

$$X(0) = \sum_{k=0}^{3} x(k) = x(0) + x(1) + x(2) + x(3) = 1 + 1 + 0 + 2 = 4$$

$$k=0$$

$$=1+e^{-j\pi/2}+2e^{-j3\pi/2}=1+(\cos\pi/2-j\sin\pi/2)+2(\cos3\pi/2-j\sin3\pi/2)$$

$$=1-j+2j=1+j$$

$$X(2)=\sum_{k=0}^{3}x(k)e^{-j\pi k}=x(0)+x(1)e^{-j\pi}+x(2)e^{-j2\pi}+x(3)e^{-j3\pi}$$

$$=1+e^{-j\pi}+2e^{-j3\pi}=1+(\cos\pi-j\sin\pi)+2(\cos3\pi-j\sin3\pi)$$

$$=1-1-2=-2$$

$$X(3)=\sum_{k=0}^{3}x(k)e^{-j3\pi k/2}=x(0)+x(1)e^{-j3\pi/2}+x(2)e^{-j3\pi}+x(3)e^{-j9\pi}/2$$

$$=1+e^{-j3\pi/2}+2e^{-j9\pi/2}=1+(\cos3\pi/2-j\sin3\pi/2)+2(\cos9\pi/2-j\sin9\pi/2)$$

$$=1+j-2j=1-j$$

i.e Time domain $N=4 \Rightarrow X(n)=[1,1,0,2]$ &frequency domain i.e $K=4 \Rightarrow X(K)=[4,1+j,-2,1-j]$

the time soltion of 2 point usin DFT IS 7 min while for FFT is14 min

$$N=8 \Rightarrow X(n)=[1,2,1,0,1,3,1,2]$$

$$X(n)=\sum_{k=0}^{7}x(k)e^{-j\pi nk/4} \qquad (13)$$

$$X(0)=\sum_{k=0}^{7}x(k)=x(0)+x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)$$

$$=1+2+1+0+1+3+1+2=11$$

$$X(1)=\sum_{k=0}^{7} x(k)e^{-j\pi k/4}=x(0)+x(1)e^{-j\pi/4}+x(2)e^{-j\pi/2}$$

$$+x(3)e^{-j\pi 3/4}+x(4)e^{-j\pi}+x(5)e^{-j\pi 5/4}+x(6)e^{-j\pi 3/2}+x(7)e^{-j7\pi/4}$$

$$=1+2(\cos\pi/4-j\sin\pi/4)+(\cos\pi/2-j\sin\pi/2)$$

$$+(\cos\pi-j\sin\pi)+3(\cos 5\pi/4-j\sin\pi 5/4)$$

$$+(\cos 3\pi/2-j\sin 3\pi/2)+2(\cos 7\pi/4-j\sin 7\pi/4)=-0.7071+2.1213j$$

$$X(2)=\sum_{K=0}^{7} x(k)e^{-j\pi k/2}=x(0)+x(1)e^{-j\pi/2}+x(2)e^{-j\pi}+x(3)e^{-j3\pi/2}$$

$$+x(4)e^{-j2\pi}+x(5)e^{-j5\pi/2}+x(6)e^{-j3\pi}+x(7)e^{-j7\pi/2}=-3j$$

$$X(3)=\sum_{K=0}^{7} x(k)e^{-j3\pi k/4}=x(0)+x(1)e^{-j3\pi/4}+x(2)e^{-j\pi/2}$$

$$+x(3)e^{-j9\pi/4}+x(4)e^{-j3\pi}+x(5)e^{-j5\pi/2}+x(6)e^{-j3\pi}+x(7)e^{-j21\pi/4}$$

$$=-0.7071+2.1213j$$

$$X(4)=\sum_{k=0}^{7} x(k)e^{-j\pi k}=x(0)+x(1)e^{-j\pi}+x(2)e^{-j2\pi}+x(3)e^{-j3\pi}$$

$$+x(4)e^{-j4\pi}+x(5)e^{-j5\pi}+x(6)e^{-j6\pi}+x(7)e^{-j7\pi}=-3$$

$$X(5)=\sum_{k=0}^{7} x(k)e^{-j5\pi k/4}=x(0)+x(1)e^{-j5\pi/4}+x(2)e^{-j5\pi/2}+x(3)e^{-j15\pi/4}$$

$$+x(4)e^{-j4\pi}+x(5)e^{-j25\pi/4}+x(6)e^{-j15\pi/2}+x(7)e^{-j35\pi/4}$$

$$=-0.7071-2.1213j$$

$$X(6)=\sum_{k=0}^{7}x(k)e^{-j3\pi k/2}=x(0)+x(1)e^{-j3\pi/2}+x(2)e^{-j3\pi}+x(3)e^{-j9\pi/2}$$

$$+x(4)e^{-j6\pi}+x(5)e^{-j5\pi/2}+x(6)e^{-j9\pi}+x(7)e^{-j21\pi/2}=3j$$

$$X(7)=\sum_{k=0}^{7}x(k)e^{-j7\pi k/4}=x(0)+x(1)e^{-j7\pi/4}+x(2)e^{-j7\pi/2}+x(3)e^{-j21\pi/4}$$

$$+x(4)e^{-j7\pi}+x(5)e^{-j35\pi/4}+x(6)e^{-j21\pi/2}+x(7)e^{-j49\pi/4}$$

$$=0.7071-2.1213j$$

i.e Time domain $N=8 \Rightarrow X(n)=[1,2,1,0,1,3,1,2]$

&frequency domain

$$K=4 \Rightarrow X(K)=\begin{bmatrix}11,0.7071+2.1213j,-3j,-0.7071+2.1213j \\ -3,-0.7071-2.1213j,3j,0.7071-2.1213j\end{bmatrix}$$

the time soltion of 2 point usin DFT IS 15 min while for FFT is30 min

$N=16 \qquad \Rightarrow [1,2,3,1,3,2,4,6,2,5,3,7,6,3,1,0]$

$$X(n)=\sum_{k=0}^{15}x(k)e^{-j\pi nk/8} \quad (14)$$

$$X(0)=\sum_{k=0}^{15}x(k)=49$$

$$X(1)=\sum_{k=0}^{15}x(k)e^{-j\pi k/8}=-13.3497+6.1978j$$

$$X(2)=\sum_{k=0}^{15}x(k)e^{-j\pi k/4}=-6-1.8284j$$

$$X(3)=\sum_{k=0}^{15} x(k)e^{-j3\pi k/8}=2.2966-10.5717j$$

$$X(4)=\sum_{k=0}^{15} x(k)e^{-j\pi k/2}=1+2j$$

$$X(5)=\sum_{k=0}^{15} x(k)e^{-j5\pi k/8}=-0.0539-0.3291j$$

$$X(6)=\sum_{k=0}^{15} x(k)e^{-j3\pi k/4}=-6-1.8284j$$

$$X(7)=\sum_{k=0}^{15} x(k)e^{-j7\pi k/8}=7.1070+4.4404j$$

$$X(8)=\sum_{k=0}^{15} x(k)e^{-j\pi k}=-3$$

$$X(9)=\sum_{k=0}^{15} x(k)e^{-j9\pi k/8}==7.070-4.4404j$$

$$X(10)=\sum_{k=0}^{15} x(k)e^{-j5\pi k/4}=-6+1.8284j$$

$$X(11)=\sum_{k=0}^{15} x(k)e^{-j11\pi k/8}=-0.0539+03291j$$

$$X(12)=\sum_{k=0}^{15} x(k)e^{-j3\pi k/2}=1-2j$$

$$X(13)=\sum_{k=0}^{15} x(k)e^{-j13\pi k/8}=2.2966+10.5717j$$

$$X(14)=\sum_{k=0}^{15} x(k)e^{-j7\pi k/4}=-6+3.8284j$$

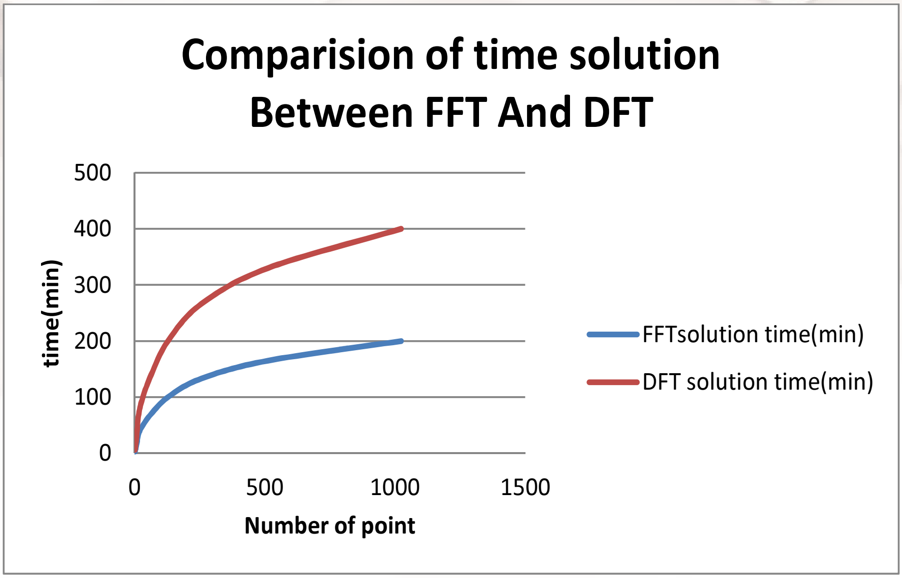
$$X(15)=\sum_{k=0}^{15} x(k)e^{-j15\pi k/8}=-13.3497-6.1978j$$

By the same process FFT for number of point of 32,64 ,128 ,256 ,512 ,1024 can be calculated but take long time

3- Results:

After execution of software program using excel the following results were obtained to compare the solutionbetween DFT and FFT

Number of point	FFTsolution time(min)	DFT solution time(min)
2	3	6
4	7	14
8	15	30
16	35	70
32	50	100
64	70	140
128	100	200
256	133	266
512	165	330
1024	200	400



Figure(4) : comparison of time solution between FFT and DFT

Results Discussion:

Fig(4) shows comparison of time solution between FFT and DFT against the number of point .from the fig we observe that as the number of point increase the solution time increased and the average time for solving one point using discrete fourier transform is 0.76050 min. while the average solution time using FFT is 0.38025 the saved in time by using FFT is 0.38025while the FFT solution time using MATLAB is real time solution in the instant of command

8-Conclusion:

The study, analyize and design software program to measure and plot the execution time of DFT and FFT have been done using excel software program. The parameter which were taken intoConsideration for analysis is number of point, type of FFT, the results show that the average time for solving one point using discrete fourier transform is 0.76050 min. while the average solution time using FFT is 0.38025min the FFT reduce the solution time 0.38025min .

Recommendation:

From the results obtained we suggest the following Recommendation for future work:

- To increase the order of number point for DFT and FFT and find the solutiontime.

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