MODIFIED BISECTION METHOD

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Abstract:

The aim of the research is to compare the method of bisection and the method of modified bisection, to solve non-linear equations by numerical methods to find the root of the equation, through definitions, theories and examples, and we conclude through that that to solve non-linear equations we use the modified bisection method directly because it converges with the values of the root quickly and accurately and with the least number of iterations and also It is characterized by its validity and applicability to all categories of problems in non-linear equations. As for the bisection method, its advantage is its reliability, but the disadvantage of this method is that it takes a number of iterations compared to the modified bisection method, whose significance stems from the fact that it saves time and effort.best.

Keywords: Nonlinear equation , Root-finding problem, Bisection method, , Bisection Convergence and Error Theorem, modified bisection method .

طريقة التنصيف المعدلة

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المستخلص:

الهدف من البحث هوالمقارنة بين طريقة التنصيف و طريقة التنصيف المعدلة ,لحل المعادلات الغير خطيةبالطرق العددية لايجاد جذرالمعادلة , وذلك من خلال التعاريف والنظريات والامثلة ونستنتج من خلال ذلك انه لحل المعادلات الغير خطية نستخدم طريقةالتنصيف المعدلةمباشرةً لأنها تتقارب مع قيم الجذر بسرعة ودقه و باقل عدد تكرارات وايضا تتميز بصلاحيتها وقابليتها للتطبيق على جميع فئات المشاكل في المعادلات الغير الخطية اما طريقة التنصيف ميزتها هي موثوقيتها ولكن عيب هذةالطريقة هو أنها تأخذ عددًا من التكرارات مقارنة بطريقة التنصيف المعدلة التي تكمن اهميتها في اختصارها للوقت والجهد .

الكلمات المفتاحية : المعادلة الغير خطية , مشكلة إيجاد الجذور , طريقة التنصيف , التقارب ونظرية الخطأ , طريقة التنصيف المعدلة.

1. INTRODUCTION

The root finding problem is a basic numerical analysis problem t t should be expressed as a root that satisfies the provided equation$f(t) = 0f(t) = 0$. One of the main goals of numerical analysis is to create a numerical algorithm for estimating values. Newton's method, secant method, fixed point iteration method, and bisection method are all well-known root finding method [1,2][1,2],in numerical analysis.

In Newton's case an given initial value $I_0, a\, I_0, a$ sequence $\{I_n\}_{n=1}^{\infty}\{I_n\}_{n=1}^{\infty}$is generated by

$$I_n = I_{n-1} - \frac{f(I_{n-1})}{f^{`}(I_{n-1})}, I_n = I_{n-1} - \frac{f(I_{n-1})}{f^{`}(I_{n-1})}, \text{ for } n \geq 1 n \geq 1$$

converges on a single point Because derivatives might be difficult to obtain, the secant technique, which uses the average rate of change instead of derivatives, can be employed instead of the Newton method. A root finding technique of $f(t)f(t)$utilizing the form of $g(t) = t(t) = t$ is also a fixed point iteration approach in which a sequence is produced so that it converges to a root.

One of the root-finding methods is the bisection method

[3,4,5,6,7,8,9][3,4,5,6,7,8,9]. Cut the interval in half and use the Intermediate Value Theorem [10,11][10,11] to find the little sub-interval with the root. Although the bisection method only employs two sub-intervals, it is possible to set more than two. The Intermediate Value Theorem, in other words, is applied to three or more sub-intervals. Section 5 explains the contents of this method. Convergence and error bound are also discussed. It is also feasible to estimate the number of iterations that will be visualized in a graph using the bisection approach.

Theorem 1.1. (Intermediate Value Theorem): Given a continuous real valued function $f(x)f(x)$ defined on an interval [a, b], then if y is a point between the values of $f(a) and\ f(b)\ f(a) and\ f(b)$, then there exists a point $\mu\mu$ such that $= f(\mu) = f(\mu)$.

2- NONLINEAR EQUATION

We will considere any one of the following types:

I-A polynomial equation of degree n:

$$c_m t^m + c_{m-1} t^{m-1} + .. + c_1 t \quad + c_0 = 0$$

$$c_m t^m + c_{m-1} t^{m-1} + .. + c_1 t \quad + c_0 = 0$$
.

, where $c_m, c_m,\quad c_{m-1}, .., c_1, c_{m-1}, .., c_1$, and $c_0 c_0$ are constants. For example, the following equations are nonlinear

$$t^4 + 11t^3 - 5t + 7 = 0 \quad , \quad t^2 + 3t - 1 = 0$$

$$t^4 + 11t^3 - 5t + 7 = 0 \quad , \quad t^2 + 3t - 1 = 0$$
.

II-The power of the unknown variable (not a positive integer number). For example, the following non-polynomial equations are nonlinear

$$t^{-2} + 3t = 6 \quad , t^{\frac{4}{3}} + 8t = 0 \ , \sqrt{t} - 5t = 0$$

$$t^{-2} + 3t = 6 \quad , t^{\frac{4}{3}} + 8t = 0 \ , \sqrt{t} - 5t = 0$$
.

III- The equation which involves the trigonometric functions, exponential functions and logarithmic functions. For example, all the following transcendental equations are nonlinear

$$sin\, sin\, t + 2t = 0 \ , \quad e^t + t = 2 \ , 3t + ln\, t = 0$$

$sin\, sin\, t + 2t = 0$, $e^t + t = 2$,$3t + ln\, t = 0$.

Definition 2.1 Roots of nonlinear equations

One of the most basic problems that numerical techniques are used for is finding roots of nonlinear equations. This process involves finding a root, or solution, of an equation of the form

$f(t) = 0 \quad (1)$

A root of this equation is also called a zero of the function ff. Here, we are going to look at some common numerical methods for finding roots of equations

3-The Bisection Method

Definition 3.1 The Bisection Method

The Bisection method is used to determine, to any specified accuracy that your computer will permit, a solution to $f(t)f(t) = 0$ on an interval $[\alpha, \beta][\alpha, \beta]$, provided

*$f(t)f(t)$is continuous on $[\alpha, \beta][\alpha, \beta]$

* $f(\alpha)$* $f(\alpha)$ and $f(\beta)f(\beta)$ are of opposite sign.

The concept of the Bisection method is simple, and is based on utilizing the Intermediate Value Theorem. Essentially, due the continuity of g on $[\alpha, \beta][\alpha, \beta]$, and since

** $f(\alpha)f(\beta)f(\alpha)f(\beta) < 0$, then there must be a point $\alpha < \tau\tau < \beta\, \beta$ such that $f(\tau)f(\tau) = 0$. The implication is that one of the values is negative and the other is positive. These conditions can be easily satisfied by sketching the function. Therefore the root must lies between $\alpha\alpha$ and $\beta\beta$ (by Intermediate Value Theorem) and a new approximation to the root $\tau\tau$ be calculated as

$$c = \frac{\alpha + \beta}{2} c = \frac{\alpha + \beta}{2}$$

, and, in general

$$c_m = \frac{\alpha_m + \beta_m}{2} c_m = \frac{\alpha_m + \beta_m}{2}, \qquad m \geq 1 \qquad (2)$$

If $f(c)f(c) \approx 0$, then $c \approx \tau\tau$ is the desired root, and, if not, then there are two possibilities

Firstly, if $f(\alpha)f(c)f(\alpha)f(c) < 0$, then $f(t)f(t)$ has a zero between point $\alpha\alpha$ and point c. The process can then be repeated on the new interval $[\alpha, c][\alpha, c]$

Secondly, if $f(\alpha)f(c)f(\alpha)f(c) > 0$ it follows that $f(\beta)f(c)$ $f(\beta)f(c) < 0$ since it is known that $f(\beta)f(\beta)$ and $f(c)f(c)$ have opposite signs. Hence, $f(t)f(t)$ has zero between point c and point $\beta\beta$ and the process can be repeated with $[c, \beta][c, \beta]$. We see that after one step of the process, we have found either a zero or a new bracketing interval which is precisely half the length of the original one.

The process continue until the desired accuracy is achieved..

Example 3.2

Use the bisection method to find the approximation to the root of the equation

$t^3 = 2t + 1t^3 = 2t + 1$, that is located in the interval [1.5, 2.0] accurate to within$10^{-2}10^{-2}$.

Solution.

Since the given function $g(t) = t^3 - 2t - 1\ (t) = t^3 - 2t - 1$ is a polynomial function and so is continuous

on [1.5, 2.0], starting with a_1a_1= 1.5 and b_1b_1= 2, we compute: f(a_1a_1) = −0.625 , f(b_1b_1) = 3.0, and since g(1.5)g(2.0) < 0, so that a root of g(t) = 0 lies in the interval [1.5, 2.0]

Using formula (2) (when n = 1), we get:

$$\tau_1 = \frac{a_{1+}\, b_1}{2} = 1.75$$

g ($\tau_1\tau_1$) = 0.859375.

Hence the function changes sign on [a_1a_1, $\tau_1\tau_1$] = [1.5, 1.75]. To continue, we squeeze from right and set a_2a_2=a_1a_1 and b_2b_2=τ_1 τ_1.

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Then the midpoint is:

$\tau_2 = \frac{a_2 + b_2}{2} \tau_2 = \frac{a_2 + b_2}{2} = 1.625$, $g(\tau_2 \tau_2) = 0.041056.$

Continue in this way we obtain a sequence $\tau_k \tau_k$ of approximation shown by table

N	Left Endpoint a_n a_n	Midpoint τ_n τ_n	Right Endpoint b_n b_n	Function Value $g(\tau_n)\tau_n)$
1	1.500000	1.750000	2.0000000	0.8593750
2	1.500000	1.625000	1.750000	0.0410156
3	1.500000	1.625000	1.625000	-0.3103027
4	1.562500	1.593750	1.625000	-0.1393127
5	1.593750	1.609375	1.625000	-0.0503273
6	1.609375	1.617188	1.625000	-0.0049520

Table(1)

We see that the functional values are approaching zero as the number of iterations is increase. We got the desired approximation to the root of the given equation is

$\tau_6 \tau_6 = 1.617188$ after 6 iterations with accuracy $\in = 10^{-2}$ $\in = 10^{-2}$.

Example 3.3

$g(t) = cost - t(t) = cost - t$.

The results of the problem g $(t) = cost - t = 0$ $(t) = cost - t = 0$ is obtained by using the Bisection Method. The initial interval is chosen as [0,1]. The iteration number and the τ τ value obtained after each iteration are provided as follows:

Iteration Number	τ_n
1	0.5
2	0.75
3	0.625
4	0.6875
5	0.71875
6	0.734375
7	0.7421875
8	0.73828125
9	0.740234375
10	0.7392578125
11	0.73876953125
12	0.739013671875
13	0.7391357421875
14	0.73907470703125
15	0.739105224609375

Table(2)

Table (2): The results for Bisection Method . The absolute value of the error is again smaller than the desired value 0.0001. That is the input for the applet by the user.

Example 3.4

$g(t) = e^3 - 8$

The results of the problem $g(t) = e^3 - 8 = 0$ $g(t) = e^3 - 8 = 0$ is obtained by using the Bisection Method.

The initial interval is chosen as [0,3]. The iteration number and the τ τ value obtained after each iteration are provided as follows:

Iteration Number	τ_n
1	1.5
2	0.75
3	1.125
4	1.3125
5	1.21875
6	1.265625
7	1.2890625
8	1.27734375
9	1.271484375
10	1.2744140625
11	1.27587890625
12	1.276611328125
13	1.2762451171875
14	1.27642822265625
15	1.276336669921875
16	1.2763824462890625

Table3
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Table3: The results for Bisection Method

The absolute value of the error is again smaller than the desired value 0.0001.

4- Bisection Convergence and Error Theorem

Theorem4.1

Let $f(t)f(t)$ be continuous function defined on the given initial interval $[\alpha_0,\beta_0][\alpha_0,\beta_0]$=$[\alpha,\beta][\alpha,\beta]$ and suppose that $f(\alpha)f(\beta)$ $f(\alpha)f(\beta)$< 0. Then bisection method (2) generates a sequence $\{c_m\}_{m=1}^{\infty}\{c_m\}_{m=1}^{\infty}$1 approximating $\tau \in (\alpha\,.\,\beta)\tau \in (\alpha\,.\,\beta)$with the proport

$$|\tau - c_n| \leq \frac{\beta-\alpha}{2^n} \quad ,n \geq 1 \qquad (3)$$

Example 4.2

Use the bisection method to compute the first three approximate values for $\sqrt[4]{18}\sqrt[4]{18}$ Also, compute an error bound and absolute error for your approximation.

Solution.

Consider

$t = \sqrt[4]{18} \;\; or \;\; t^4 - 18 = 0$

Choose the interval [2, 2.5] on which the function g$(t) = t^4 - 18$ $(t) = t^4 - 18$ is continuous and the function g$(t)(t)$ satisfies the sign property, that is

g(2)g(2.5) = (−2)(21.0625) = −42.125 < 0

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Hence root $\boldsymbol{\tau} = \sqrt[4]{\mathbf{18}}\;\; \boldsymbol{\tau} = \sqrt[4]{\mathbf{18}}$ = 2.0598 ∈ [2, 2.5] and we compute its first approximate value by using formula (2) (when n = 1) as follows:

$\boldsymbol{c_1} = \frac{\mathbf{2+2.5}}{\mathbf{2}} = \boldsymbol{c_1} = \frac{\mathbf{2+2.5}}{\mathbf{2}} =$ 2.2500 and g(2.25) = 7.6289.

Since the function $\mathbf{g}(\boldsymbol{t})(\boldsymbol{t})$ changes sign on [2.0, 2.25]. To continue, we squeeze from right and use formula (2) again to get the following second approximate value of the root $\boldsymbol{\tau\tau}$ as:

$\boldsymbol{c_2} = \frac{\mathbf{2+2.25}}{\mathbf{2}} = \boldsymbol{c_2} = \frac{\mathbf{2+2.25}}{\mathbf{2}} =$ 2.1250 and g(2.1250) = 2.3909.

Then continue in the similar way, the third approximate value of the root $\tau\tau$ is

c_3c_3 = 2.0625 with g(2.0625) = 0.0957

Note that the value of the function at each new approximate value is decreasing which shows that the approximate values are coming closer to the root $\tau\tau$. Now to compute the error bound for the approximation we use the formula (3) and get

$|\,\tau\tau - c_3c_3| \leq \frac{2.5-2 \; 2.5-2}{2^3 \quad 2^3} = 0.0625,$

which is the possible maximum error in our approximation and

|E| = |2.0598 − 2.0625| = 0.0027

be the absolute error in the approximation.

5-MODIFIED BISECTION METHOD

One of the ways for obtaining the root of the equation $f(t) = 0$ $f(t) = 0$is the bisection approach. Assume f is a continuous function defined on the interval$[\alpha, \beta][\alpha, \beta]$ with opposite-sign $f(\alpha) and\, f(\beta) f(\alpha) and\, f(\beta)$This approach finds a sub-interval satisfying the Intermediate Value Theorem by cutting half of the interval $[\alpha, \beta][\alpha, \beta]$Cut the half again after re-initializing the specified sub-interval at both ends. When this technique is repeated, the error bound becomes as narrow as the interval of the previous iteration.

The first three stage of the bisection approach are shown in Figure 1, with I_3 I_3 being produced in the third iteration with the $\frac{\beta_3-\alpha_3}{2}$ $\frac{\beta_3-\alpha_3}{2}$ error bound. As a result, if the bisection approach is used repeatedly, as shown in Figure 1, the estimated value will eventually converge to a root.

The mid-point is used as an approximate value in the bisection method. If we look at it from a different perspective, we can see that it is made up of two evenly spaced sub-intervals. If we define $h\, as \frac{\beta\ -\alpha}{2}$ $h\, as \frac{\beta\ -\alpha}{2}$ in the interval $[\alpha, \beta][\alpha, \beta]$, for example, the two little sub-intervals are defined as follows: $[\alpha, \alpha + h]$,

$[\alpha, \alpha + h]$,
$[\alpha + h\,, \beta][\alpha + h\,, \beta]$ We use $\alpha + h\ \alpha + h$ as the estimated value in the bisection approach since the middle value is an approximation. The interval $[\alpha, \beta][\alpha, \beta]$ is therefore made up of three equally spaced sub-intervals with the increment h=$\frac{\beta\ -\alpha}{3}\frac{\beta\ -\alpha}{3}$,

yielding $[\alpha, \alpha + h],\ [\alpha + h\,, \alpha + 2h],[\alpha + 2h\,, \beta]$
$[\alpha, \alpha + h],\ [\alpha + h\,, \alpha + 2h],[\alpha + 2h\,, \beta]$

1

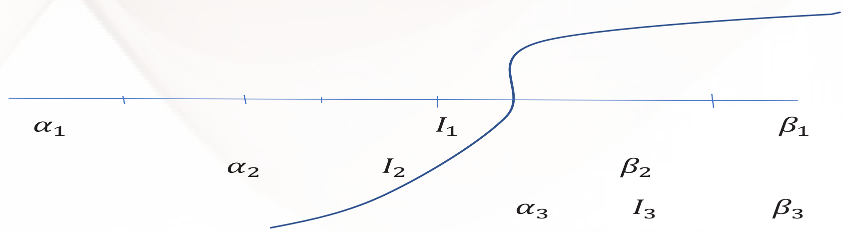

Figure 1 :Graph of the bisection method

It is feasible to determine whether a root exists in a sub-interval using the Intermediate Value Theorem for each sub-interval. Then, to further redefine new$h\ h$, identify the interval in which the root is present and define it as $\alpha\ and\ \beta\alpha\ and\ \beta$. For each sub-interval, the Intermediate Value Theorem can be utilised.
The length of the sub-interval in the last iteration then guarantees the error. We can get a better approximation using this method.
When solving a root-finding problem, it's critical to understand the convergence. Because if an approximation does not converge at a given value, it is impossible to know if it is truly meaningful as a value, we prove convergence and an error bound for the approach, demonstrating that when the number of iterations is big enough, the approximation converges to a true value.

Theorem 5.1

Let f be a continuous function and defined on $[\alpha, \beta]\,[\alpha, \beta]$ which $f(\alpha).f(\beta)f(\alpha).f(\beta)$< 0. The modified bisection method generates a sequence$\{I_n\}_{n=1}^{\infty}\{I_n\}_{n=1}^{\infty}$ with

$\alpha_k < I_k \ < \beta_k \quad for \ \ k \geq 1$

$\alpha_k < I_k \ < \beta_k \quad for \ \ k \geq 1$ (4)

Proof: Since $f(\alpha).f(\beta)f(\alpha).f(\beta)< 0$, hence we separate to two cases:

Case1: $f(\alpha_k) < 0 \ and \quad f(\beta_k) > 0 f(\alpha_k) < 0 \ and \quad f(\beta_k) > 0$

Consider a subinterval $(\alpha_k^* \ ,\beta_k^*)(\alpha_k^* \ ,\beta_k^*)$, $c_k \, c_k$ from equations (2)

(i.) If $f(\alpha_k)\, f(c_k) < 0 f(\alpha_k)\, f(c_k) < 0$ then we have $\alpha_k^* \alpha_k^* = \alpha_k \alpha_k$, $\beta_k^* \beta_k^* = c_k c_k \quad and \quad f(\beta_k) > 0 and \quad f(\beta_k) > 0$ So,

We have $f(\beta_k^*) \ . \frac{\beta_k^* - \alpha_k^*}{f(\beta_k^*) - f(\alpha_k^*)} > 0 f(\beta_k^*) \ . \frac{\beta_k^* - \alpha_k^*}{f(\beta_k^*) - f(\alpha_k^*)} > 0$

Then

$$I_k \ = \beta_k^* - f(\beta_k^*) \ . \frac{\beta_k^* - \alpha_k^*}{f(\beta_k^*) - f(\alpha_k^*)} < \beta_k^* < \beta_k$$

$I_k \ = \beta_k^* - f(\beta_k^*) \ . \frac{\beta_k^* - \alpha_k^*}{f(\beta_k^*) - f(\alpha_k^*)} < \beta_k^* < \beta_k$

Since

$$f(\alpha_k^*) \ . \frac{\beta_k^* - \alpha_k^*}{f(\beta_k^*) - f(\alpha_k^*)} < 0$$

then, we have that

$$I_k \ = \ \alpha_k^* - f(\beta_k^*) \ . \frac{\beta_k^* - \alpha_k^*}{f(\beta_k^*) - f(\alpha_k^*)} > \ \alpha_k^* = \alpha_k$$

Hence,

$\alpha_k < I_k \ < \beta_k$

ii) if

$f(c_k) f(\beta_k) < 0 \ \ then \, we \, have \ \ \alpha_k^* = c_k, \ \beta_k^* = \beta_k \ \ and f(\alpha_k^*) < 0$

$f(c_k) f(\beta_k) < 0 \ \ then \, we \, have \ \ \alpha_k^* = c_k, \ \beta_k^* = \beta_k \ \ and f(\alpha_k^*) < 0$

The proof is similarly.

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Case2: $f(\alpha_k) > 0 \ and \quad f(\beta_k) < 0 f(\alpha_k) > 0 \ and \quad f(\beta_k) < 0$

This proof is rather similar to the above .

Theorem 5.2

Suppose that f is a continuous function on $[\alpha,\beta][\alpha,\beta]$ and $f(\alpha)f(\beta)<0 f(\alpha)f(\beta)<0$. Assume the interval is divided into equally spaced s sub-intervals. Then the sequence $\{I_n\}_{n=1}^{\infty}\{I_n\}_{n=1}^{\infty}$ approximates a zero of $f\ whith\ \ |I_n-I|\le\frac{\beta-\alpha}{s^n}\ f\ whith\ \ |I_n-I|\le\frac{\beta-\alpha}{s^n}$ (5) .

Proof . Let $\alpha_1=\alpha\ ,\ \beta_1=\beta\ \ and\ set\ h_n=\frac{\beta_n-\alpha_n}{s}$ $\alpha_1=\alpha\ ,\ \beta_1=\beta\ \ and\ set\ h_n=\frac{\beta_n-\alpha_n}{s}$ for each $n\ge1 n\ge1$.

We can pick approximatioms

$I_{n_1}I_{n_1}=\alpha_n+h_n\alpha_n+h_n$, I_{n_2} , $I_{n_2}=\alpha_n+2h_n\alpha_n+2h_n$,…………, $I_{n_k}I_{n_k}=\alpha_n+kh_n\alpha_n+kh_n$, Since k= s-1

$I_{n_k}I_{n_k}=$

$$\alpha_n+kh_n=\alpha_n+k\left(\frac{\beta_n-\alpha_n}{s}\right)=\alpha_n+(s-1)\left(\frac{\beta_n-\alpha_n}{s}\right)=$$

$$\alpha_n+kh_n=\alpha_n+k\left(\frac{\beta_n-\alpha_n}{s}\right)=\alpha_n+(s-1)\left(\frac{\beta_n-\alpha_n}{s}\right)=$$

$$\frac{1}{s}\ \alpha_n+\left(1-\frac{1}{s}\right)\beta_n$$

Since

$$\alpha_n\le I\le\beta_n\ ,\alpha_n-\left(\frac{1}{s}\ \alpha_n+\frac{s-1}{s}\beta_n\right)\le I-I_{n_k}\le\beta_n-\left(\frac{1}{s}\ \alpha_n+\frac{s-1}{s}\beta_n\right.$$

$$\alpha_n\le I\le\beta_n\ ,\alpha_n-\left(\frac{1}{s}\ \alpha_n+\frac{s-1}{s}\beta_n\right)\le I-I_{n_k}\le\beta_n-\left(\frac{1}{s}\ \alpha_n+\frac{s-1}{s}\beta_n\right.$$

$$0\le(1-\frac{1}{s})(\beta_n-\alpha_n)\le I-I_{n_k}\le\frac{1}{s}(\beta_n-\alpha_n)$$

Because $\beta_n-\alpha_n=\frac{\beta_{n-1}-\alpha_{n-1}}{s}=\cdots=\frac{\beta_1-\alpha_1}{s^{n-1}}\ we\ have$

$\beta_n-\alpha_n=\frac{\beta_{n-1}-\alpha_{n-1}}{s}=\cdots=\frac{\beta_1-\alpha_1}{s^{n-1}}\ we\ have$

$$0\le I-I_{n_k}\le\frac{\beta_1-\alpha_1}{s^n}\quad for\ n\in N\ .$$

$0 \leq I - I_{n_k} \leq \frac{\beta_1 - \alpha_1}{s^n} \quad for\, n \in N\,.$

Hence $\quad I_{n_k} \to I \quad when\, n \to \infty, that\, is I_n \to I \quad when\, n \to \infty,$

$I_{n_k} \to I \quad when\, n \to \infty, that\, is I_n \to I \quad when\, n \to \infty,$

We try to estimate the number of iterations using the error bound. For given error bound ϵ, the number of iterations n satisfies the following inequality

$$\frac{\beta - \alpha}{s^n} \leq \varepsilon, \quad that\, is\, n \geq \frac{ln\, ln\, (\frac{\beta - \alpha}{\varepsilon})}{ln\, ln\, s}$$

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Where $\alpha, \beta\alpha, \beta, \epsilon, s$ are all constants in the inequality above, the number of iteration can be predicted. For the sake of simplicity, we use

$\frac{lnln\, c}{lnln\, s} \quad , where\, c = (\frac{\beta - \alpha}{\varepsilon})$

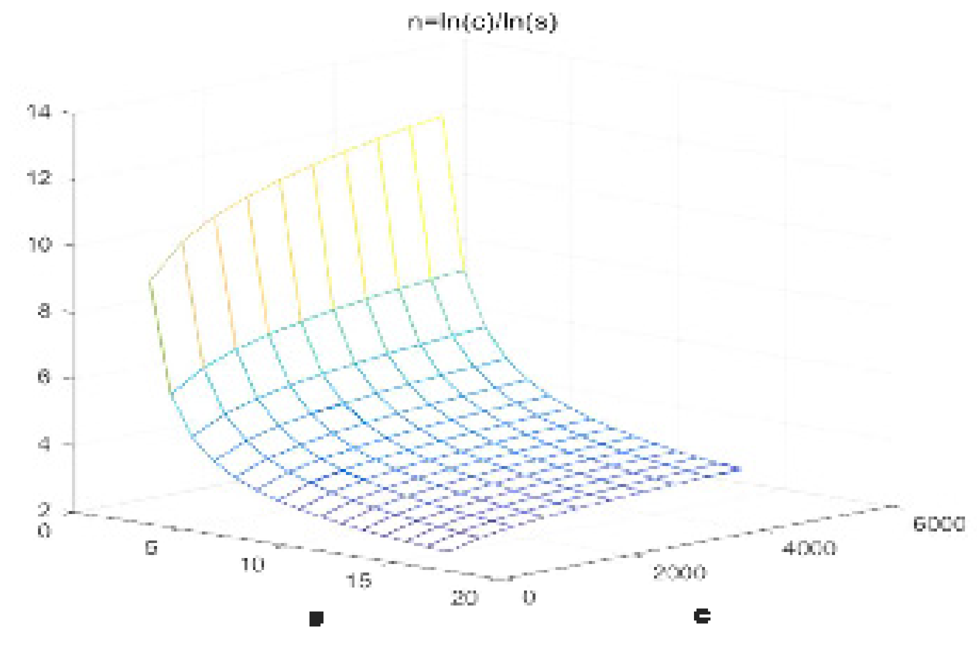

Figure 2: Diagram of the relation n $\quad$ *for various c and s*

The graph of the number of iterations for the various error bounds and the number of sub-intervals is shown in Figure 2. shows that as the number of sub-intervals grows, the number of iterations reduces. At c = 2000, for example, the graph shows that n declines as s grows. And when s is constant, n advances in perfect agreement with c.

Example5-3

Using the modified bisection method to Solve the non-linear equation

less than 0.1 × 10^{-7} 10^{-7} are below:

I. $f(t)_1 = t^2 - e^t - 3t + 2 \;\; on \; [-2,2]$

$f(t)_1 = t^2 - e^t - 3t + 2 \;\; on \; [-2,2]$

II. $f(t)_2 = t^3 - 2t + 2 \;\; on \; [-3,3]$

$f(t)_2 = t^3 - 2t + 2 \;\; on \; [-3,3]$

III. $f(t)_3 = sin\,(t) - t^5 + t^3 - 1 \;\; on \; [-5,2]$

$f(t)_3 = sin\,(t) - t^5 + t^3 - 1 \;\; on \; [-5,2]$

$f(t)$	method	Interval	No. iterations	Solution
$f(t)_1$	Bisection method	$[-2,2]$ $t_0 = 0,-1$	28	0.257530
	Modified bisection method		4	0.257530
$f(t)_2$	Bisection method	$[-3,3]$ $t_0 = 0,-1$	22	-1.769292
	Modified bisection method		9	-1.769292
$f(t)_3$	Bisection method	$[-5,2]$ $t_0 = -1.5, -1$	32	-1.345573
	Modified bisection method		5	1.345573

Table(3)

Example5-4

Using the modified bisection method to Solve the non-linear equation $3t - e^{t} = 0$ with three -intervals. that the exact solution is $t = 1.51213455\cdots$.

n	Bisection method	Modified bisection method
1	1.500000	1.500000
2	1.750000	1.500000
3	1.625000	1.500000
4	1.562500	1.500000
5	1.531250	1.512346
6	1.515625	1.512346
7	1.507813	1.512346
8	1.511719	1.512346
9	1.513672	1.512193
10	1.512695	1.512142
11	1.512207	1.512142
12	1.511963	1.512137
13	1.512085	**1.512135**
14	1.512146	
15	1.512115	
16	1.512131	
17	1.512138	
18	**1.512135**	

Table(4)

RESULTS AND DISCUSSION:

We can see that the modified bisection method is working quicker and more accurate than the bisection method in the previous examples. In example (5.2), the bisection approach revealed the root of the equation in iteration 28, whereas the modified bisection method revealed the root in iteration 4. In the second equation, the bisection method revealed the root in iteration 22, while the root was discovered in iteration 9 using the modified bisection approach, as well as in the third equation We can also see that the bisection approach found the root of the problem in iteration 32, while the modified bisection method found the same root in iteration 5, and in example (5.3), we found the root by bisection method in iteration eighteenth. The modified bisection method produced the same outcome as the bisection method in the thirteenth iteration.

CONCLUSION

Finally, it has been concluded that modified bisection method performs better than Bisection method from exactness and iterative point of view, consequently, is a decent achievement to determine roots of nonlinear equation .

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